Recent progress in B_K and ε_K in lattice QCD

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Contents

SWME Collaboration
1998 — Present

SWME Collaboration

- Seoul National University (SNU):
  Prof. Weonjong Lee
  Dr. Jon Bailey and Dr. Nigel Cundy (RA Prof.)
  10+1 graduate students.

- Brookhaven National Laboratory (BNL):
  Dr. Chulwoo Jung
  Dr. Hyung-Jin Kim (Postdoc)

- Los Alamos National Laboratory (LANL):
  Dr. Boram Yoon (Postdoc)

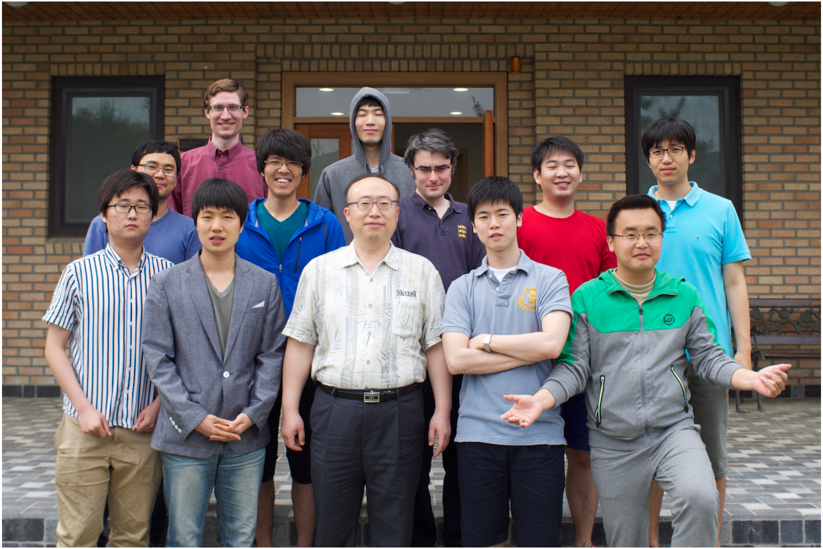
- University of Washington, Seattle (UW):
  Prof. Stephen R. Sharpe.

Lattice Gauge Theory Research Center (SNU)

- Center Leader: Prof. Weonjong Lee.
- Research Assistant Prof.: Dr. Jon Bailey
- Research Assitant Prof.: Dr. Nigel Cundy
- 10+1 graduate students
- Secretary: Ms. Sora Park.
- more details on `http://lgt.snu.ac.kr/`.

Group Photo (2011)

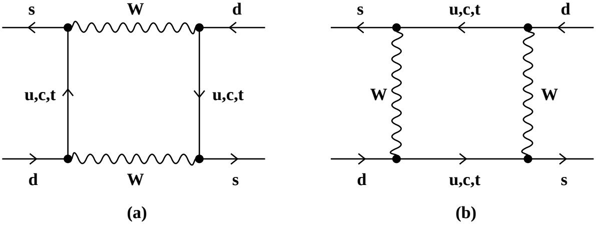
Group Photo (2013)



CP Violation and B_K

Kaon Eigenstates and ε

- Flavor eigenstates, $K^0 = (\bar{s}d)$ and $\bar{K}^0 = (s\bar{d})$ mix via box diagrams.



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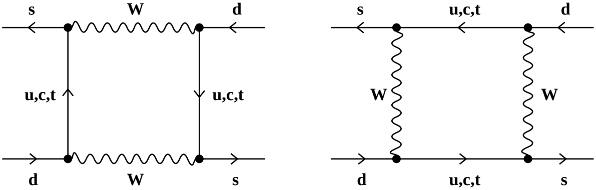

(a) (b)

- CP eigenstates K_1(even) and K_2(odd).

$$K_1 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0) \qquad K_2 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0)$$

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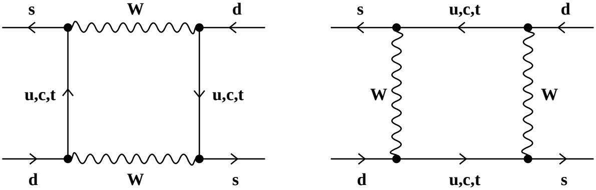

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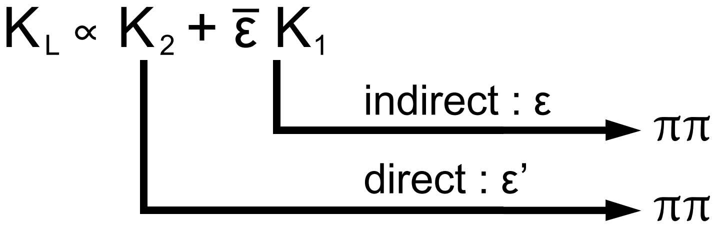
- Neutral Kaon eigenstates K_S and K_L.

$$K_S = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}(K_1 + \bar{\varepsilon} K_2) \qquad K_L = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}(K_2 + \bar{\varepsilon} K_1)$$

Indirect CP violation and direct CP violation



ε_K and $\hat{B}_K$

- Experiment: $\varepsilon_K = (2.228 \pm 0.011) \times 10^{-3} \times e^{i\phi_\varepsilon}, \quad \phi_\varepsilon = 43.52(5)^\circ$.

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\begin{aligned}
\varepsilon_K &= \exp(i\phi_\varepsilon)\ \sqrt{2}\sin(\phi_\varepsilon)\ C_\varepsilon\ \mathrm{Im}\lambda_t\ X\ \hat{B}_K + \xi + \xi_{LD} \\
X &= \mathrm{Re}\lambda_c[\eta_1 S_0(x_c) - \eta_3 S_3(x_c, x_t)] - \mathrm{Re}\lambda_t \eta_2 S_0(x_t) \\
\lambda_i &= V_{is}^* V_{id}, \qquad x_i = m_i^2/M_W^2, \qquad C_\varepsilon = \frac{G_F^2 F_K^2 m_K M_W^2}{6\sqrt{2}\pi^2 \Delta M_K} \\
\xi &= \exp(i\phi_\varepsilon)\sin(\phi_\varepsilon)\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \\
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- Definition of B_K in standard model.

$$
\begin{aligned}
B_K &= \frac{\langle \bar{K}_0 | [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] | K_0 \rangle}{\frac{8}{3}\langle \bar{K}_0 | \bar{s}\gamma_\mu\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu\gamma_5 d | K_0 \rangle} \\
\hat{B}_K &= C(\mu) B_K(\mu), \qquad C(\mu) = \alpha_s(\mu)^{-\frac{\gamma_0}{2b_0}}[1 + \alpha_s(\mu) J_3]
\end{aligned}
$$

B_K on the lattice

B_K definition in standard model

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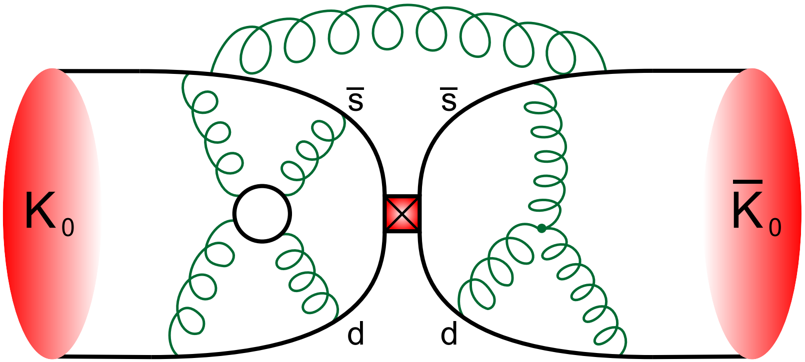
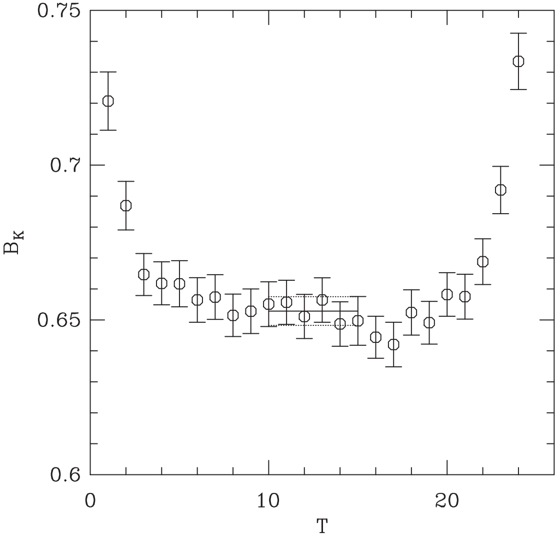
What do we calculate on the lattice?



Data Analysis for B_K

Data for B_K with $am_d = am_s = 0.025$ $(20^3 \times 64)$



$N_f = 2 + 1$ QCD: MILC coarse lattices

a (fm)	am_l/am_s	geometry	ens×meas	production
0.12	0.03/0.05	$20^3 \times 64$	564×9	done (SNU)
0.12	0.02/0.05	$20^3 \times 64$	486×9	done (SNU)
0.12	0.01/0.05	$20^3 \times 64$	671×9	done (SNU)
0.12	0.01/0.05	$28^3 \times 64$	274×8	done (BNL)
0.12	0.007/0.05	$20^3 \times 64$	651×10	done (SNU)
0.12	0.005/0.05	$24^3 \times 64$	509×9	done (SNU)

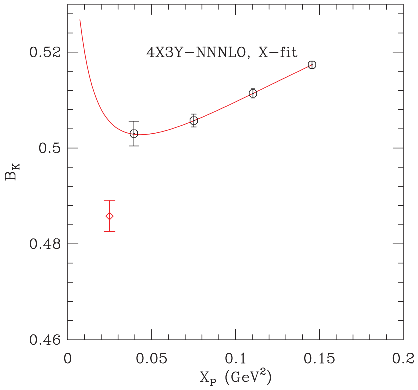
$N_f = 2+1$ QCD: MILC fine lattices

a (fm)	am_l/am_s	geometry	ens×meas	production
0.09	0.0062/0.0310	$28^3 \times 96$	995×9	done (SNU)
0.09	0.0031/0.0310	$40^3 \times 96$	959×9	done (SNU)
0.09	0.0093/0.0310	$28^3 \times 96$	950×9	done (SNU)
0.09	0.0124/0.0310	$28^3 \times 96$	1996×9	done (SNU)
0.09	0.00465/0.0310	$32^3 \times 96$	665×9	done (SNU)
0.09	0.0062/0.0186	$28^3 \times 96$	950×9	done (KISTI)
0.09	0.0031/0.0186	$40^3 \times 96$	701×9	done (SNU)
0.09	0.0031/0.0031	$40^3 \times 96$	576×9	done (KISTI)
0.09	0.00155/0.0310	$64^3 \times 96$	790×9	done (KISTI)

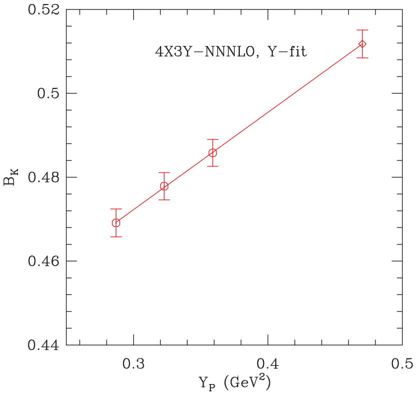
$N_f = 2 + 1$ QCD: MILC superfine/ultrafine lattice

a (fm)	am_l/am_s	geometry	ens×meas	production
0.06	0.0036/0.018	$48^3 \times 144$	744×9	done (SNU)
0.06	0.0025/0.018	$56^3 \times 144$	799×9	done (KISTI)
0.06	0.0072/0.018	$48^3 \times 144$	593×9	done (KISTI)
0.06	0.0054/0.018	$48^3 \times 144$	617×9	done (SNU)
0.06	0.0018/0.018	$64^3 \times 144$	826×7.4	(*KISTI)
0.06	0.0036/0.0108	$64^3 \times 144$	600×0.2	(*SNU)
0.045	0.0030/0.015	$64^3 \times 192$	747×1	(BNL)

Correlated Bayesian Fitting with SU(2) SChPT



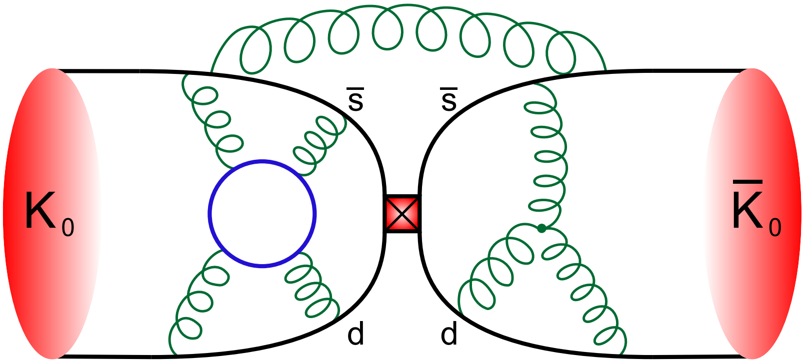
(a) X-fit



(b) Y-fit

- MILC, $48^3 \times 144$, 744 cnfs, 9 meas at $a = 0.06$ fm.

Sea quarks and valence quarks



Continuum extrapolation of B_K (1)

- Fitting functional form:

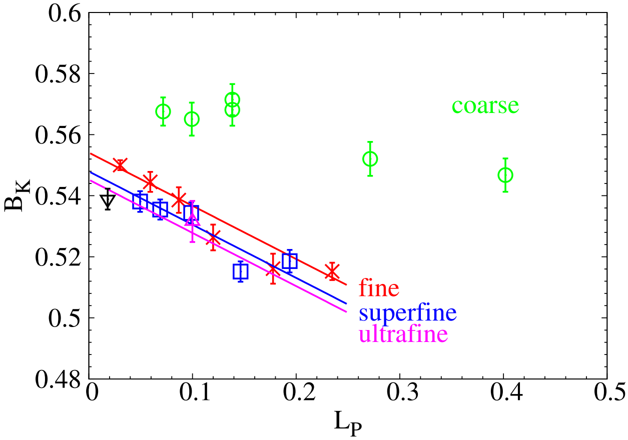
$$f_1 = c_1 + c_2(a\Lambda_Q)^2 + c_3\frac{L_P}{\Lambda_X^2} + c_4\frac{S_P}{\Lambda_X^2}$$
$$f_2 = f_1 + c_5(a\Lambda_Q)^2\frac{L_P}{\Lambda_X^2} + c_6(a\Lambda_Q)^2\frac{S_P}{\Lambda_X^2}$$
$$f_3 = f_1 + c_5\alpha_s^2 + c_6(a\Lambda_Q)^2\alpha_s + c_7(a\Lambda_Q)^4$$
$$f_4 = f_2 + c_7\alpha_s^2 + c_8(a\Lambda_Q)^2\alpha_s + c_9(a\Lambda_Q)^4$$

- Bayesian constraint = prior information:

$$\Lambda_Q = 0.3 \text{ GeV}$$
$$\Lambda_X = 1.0 \text{ GeV}$$
$$c_i = 0 \pm 2 \text{ for } i \geq 2$$

Continuum extrapolation of B_K (2)

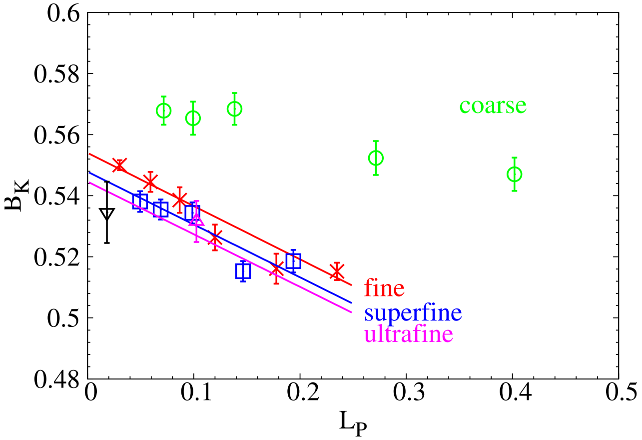
- Bayesian Fit to f_1



- We exclude the MILC coarse ensembles in this fit.

Continuum extrapolation of B_K (3)

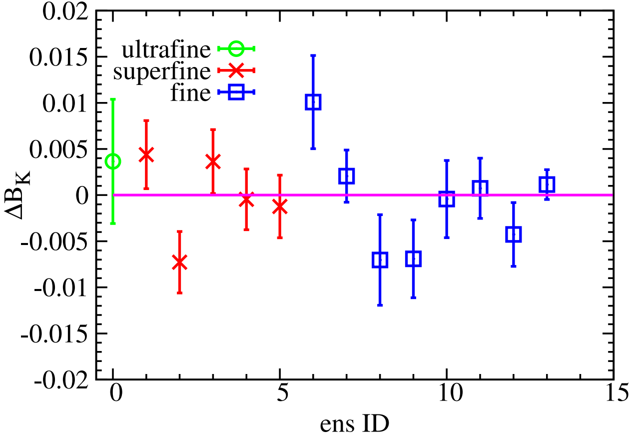
- Bayesian Fit to f_4



- We exclude the MILC coarse ensembles in this fit.

Fitting quality: ΔB_K (f_1)

- $\Delta B_K = B_K - f_1$



Continuum extrapolation of B_K (4)

fit func	f_1	f_2	f_3	f_4
χ^2/dof	1.48	1.47	1.47	1.47

Table: Fitting quality of the Bayesian fits

Error Budget of B_K [SU(2), 4X3Y, NNNLO]

cause	error (%)	memo
statistics	0.64	see text
matching factor	4.4	$\Delta B_K^{(2)}$ (U1)
{ discretization, am_ℓ extrap, am_s extrap }	0.92	diff. of B1 and B4 fits
X-fits	0.09	varying Bayesian priors
Y-fits	2.0	diff. of linear and quad.
finite volume	0.38	diff. of $V = \infty$ and FV fit
r_1	0.28	r_1 error propagation (F1)
f_π	0.10	132 MeV vs. 124.2 MeV

Current Status of B_K (1)

- SWME: 2011 (PRL):

$$B_K(\text{RGI}) = \hat{B}_K = 0.727 \pm 0.004(\text{stat}) \pm 0.038(\text{sys})$$

- SWME: 2014 (PRD)

$$B_K(\text{RGI}) = \hat{B}_K = 0.7379 \pm 0.0047(\text{stat}) \pm 0.0365(\text{sys})$$

- The statistical error remains approximately the same.
- The systematic error decrease slightly.

Current Status of ε_K

- SWME 2014: (in units of 1.0×10^{-3})

$$\varepsilon_K = 1.51 \pm 0.18 \qquad \text{for Exclusive } V_{cb}$$
$$\varepsilon_K = 1.91 \pm 0.21 \qquad \text{for Inclusive } V_{cb}$$

- Experiments:

$$\varepsilon_K = 2.228 \pm 0.011$$

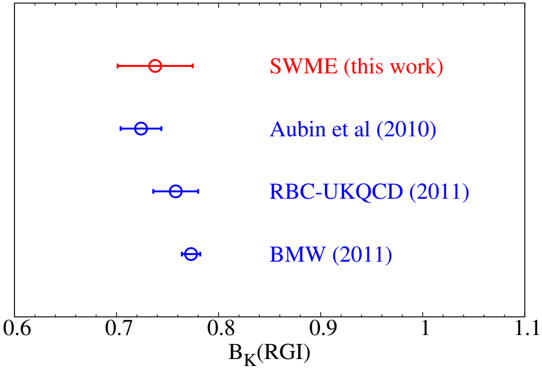
- Hence, we observe 4.0/1.5 σ difference between the SM theory and experiments (exclusive/inclusive process).

- What does this mean? $\longrightarrow$ Breakdown of SM ???

Error Budget of Exclusive ε_K

cause	error (%)	memo
V_{cb}	51.6	Exclusive (FNAL/MILC)
B_K	14.4	SWME
$\bar{\eta}$	9.7	Wolfenstein
η_3	8.1	η_{ct}
m_c	6.9	Charm quark mass
⋮	⋮	⋮

Current status of B_K on the lattice



NPR (Current Focus)

- Non-perturbative Renormalization (Jangho Kim):
  Matching factor error: 4.4% $\rightarrow$ 2.0$\sim$3.0%
- Exceptional Momentum: 2.0$\sim$3.0%
- Non-exceptional Momentum: 2.0$\sim$2.5%
- Basically, we want to trade the truncation error with the statistical error.

Two-loop Perturbation (Current Focus)

- Two-loop Perturbation: (Kwangwoo Kim)
  Matching factor error: 4.4% $\rightarrow$ 0.92%
- Automated Feynman Rule Generation.
- Automated Feynman Diagram Generation.
- Basically, we use non-zero quark masses to regulate the IR divergences.

Constraint for BSM models (Current Focus)

- Calculate the complete set of the BSM bag parameters: (95% done) $B_1 = B_K,\ B_2,\ B_3,\ B_4,\ B_5$
- Model Independent Approach to BSM models (Kwangwoo Kim)
- Model Dependent Approach to the SUSY models (Kwangwoo Kim) in collabotation with Prof. Pyungwon Ko, and Prof. Seungwon Baek.

Grand Challenges in the front

Tentative Goals (1)

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4. In addition, we need to obtain the matching factor using NPR (Jangho Kim) and using the two-loop perturbation theory (Kwangwoo Kim).
   ※ matching error $< 1.0\%$

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1. V_{cb}, we need to calculate the following semi-leptonic form factors:

$$B \to D\ell\nu \tag{1}$$

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4. We plan to work on this issue using the OK heavy quark action in collaboration with FNAL and MILC.

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3. We need $N_f = 2+1+1$ calculation on the lattice. MILC provides HISQ ensembles with $N_f = 2+1+1$.
4. As a by-product, a substantial gain is that the charm quark mass dependence might be under control in this way. (Brod and Gorbahn)

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2. As a result, we would like to provide a crucial clue to the physics beyond the standard model.
3. As a result, we would like to guide the whole particle physics community into a new world beyond the standard model.

Thank God for your help !!!